

AD-A094 281

SOUTHEASTERN MASSACHUSETTS UNIV NORTH DARTMOUTH DEPT --ETC F/6 12/1  
SOME EXPERIMENTAL RESULTS ON LINEAR ESTIMATION FOR IMAGE ANALYS--ETC(U)  
JAN 81 C H CHEN, R WU, C YEN

N00014-79-C-0494

UNCLASSIFIED

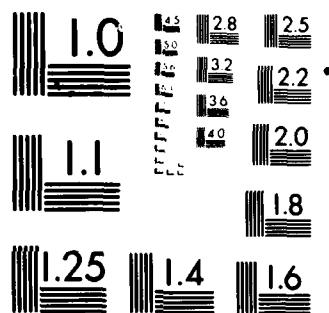
SMU-EE-TR-81-4

NL

1-1  
4/7/81



END  
DATE  
FILMED  
2 81  
DTIC



MICROCOPY RESOLUTION TEST CHART  
NATIONAL BUREAU OF STANDARDS 1963-A

# LEVEL

Technical Report  
Contract Number N00014-79-C-0494  
SMU-EE-TR-81-4  
January 28, 1981

②

AD A094281

SOME EXPERIMENTAL RESULTS ON  
LINEAR ESTIMATION FOR IMAGE ANALYSIS\*

DTIC  
ELECTE  
JAN 29 1981  
S  
C

BY  
C. H. Chen  
Rong-Hwang Wu  
Chih-sung Yen  
Department of Electrical Engineering  
Southeastern Massachusetts University  
North Dartmouth, Massachusetts 02747

\*The support of the Statistics and Probability Program of the Office of Naval Research on this work is gratefully acknowledged.

**DISTRIBUTION STATEMENT A**

Approved for public release;  
Distribution Unlimited

DDC FILE COPY

81 1 29 032

# SOME EXPERIMENTAL RESULTS ON LINEAR ESTIMATION FOR IMAGE ANALYSIS

C. H. Chen  
Rong-Hwang Wu  
Chihsung Yen

## 1. Introduction

In a previous report (ref. 1), a critical comparison is made on the various statistical image segmentation techniques. Linear estimation plays an important role in image segmentation which may be considered as an estimation problem. Included in the linear estimation methods considered in Ref. 1 are the parameter estimation for ARMA models, Fisher's linear discriminant, maximum likelihood and maximum a posteriori estimations for the regions, boundary estimation via split-and-merge algorithm, and decision-directed estimation using conditional population-mixture model. In this report, experimental results are presented for a textured subimage on the segmentation by maximum likelihood estimation, maximum a posteriori estimation, and Fisher's linear discriminant. Among the three methods, the maximum a posteriori estimation performs the best with 3.96% segmentation error, the Fisher's linear discriminant is a close second with 4.6% segmentation error and requires slightly more computation. The performance of the maximum likelihood estimation is much worse even though it requires less computation than the other two methods.

## 2. Construction of the Textured Subimage

The construction of the 64x64 textured subimage was kindly described by Dr. Charles W. Therrien of MIT Lincoln Laboratory who employed the textured subimage in his study of the linear filtering

models for image segmentation and classification (Ref. 2). The desired subimage is constructed from subimages 5 and 9 (counting from upper left to lower right horizontally) of image B2568-38 of USC data base by using the following procedure. For each point (n,m), the following expressions are evaluated.

$$B1(n, m) = (n-36) + (m-40)^2/50$$

$$B2(n, m) = ((n-48)/8)^2 + ((m-20)/12)^2 - 1$$

Then if

$B1 < 0$  or  $B2 < 0$ , take point from subimage 5

if  $B1 > 0$  and  $B2 > 0$ , take point from subimage 9

This results in a new subimage with the texture from subimage 5 above the parabolic boundary and within the ellipse and the texture from subimage 9 everywhere else. The texture from subimage 5 is denoted as type 1 while the texture from subimage 9 is denoted as type 2. Thus the segmentation is to classify for each pixel whether it is from the type 1 or from the type 2 texture. For the results in Ref. 2, the filters were 4x4 causal (first quadrant) computed by solving two-dimensional normal equations with an estimated correlation function. Transition probabilities computed in the paper made use of 5x5 neighborhoods.

By using the lineprinter display with 16 levels, Fig. 1a, 1b and 1c show respectively the original subimages 5, 9 and the constructed subimage. Two-level displays of these subimages from the Tektronix 4010 terminal are shown in Fig. 2. Fig. 3 shows the ideal segmentation as defined by the parabola and ellipse in the above equations.

|                    |                                     |
|--------------------|-------------------------------------|
| Accession For      |                                     |
| NTIS GRA&I         | <input checked="" type="checkbox"/> |
| DTIC TAB           | <input type="checkbox"/>            |
| Unannounced        | <input type="checkbox"/>            |
| Justification      |                                     |
| By                 |                                     |
| Distribution/      |                                     |
| Availability Codes |                                     |
| Avail and/or       |                                     |
| Dist Special       |                                     |

Fig. 1a  
Subimage 5

*[The page contains dense, illegible vertical text columns.]*

Fig. 1c  
Constructed  
Subimage





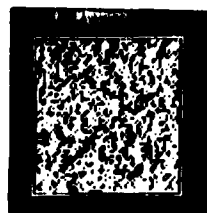


Fig. 2a Subimage 5 (above)  
in two-level display and  
histogram of the subimage  
(right). There are 16  
levels in the original  
image.

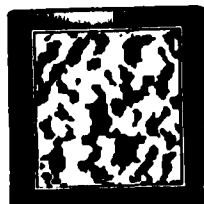
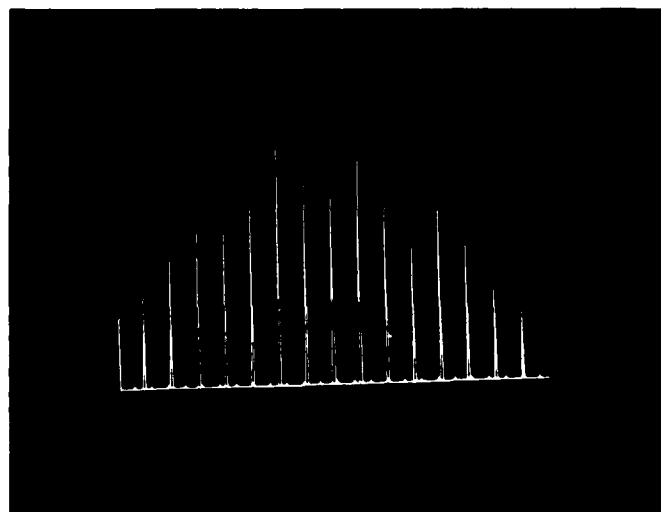


Fig. 2b Subimage 9 (above)  
in two-level display and  
histogram of the subimage  
(right).

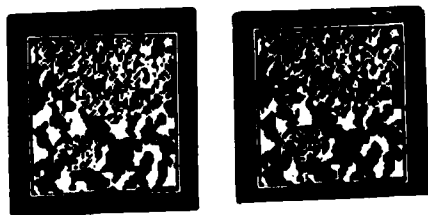
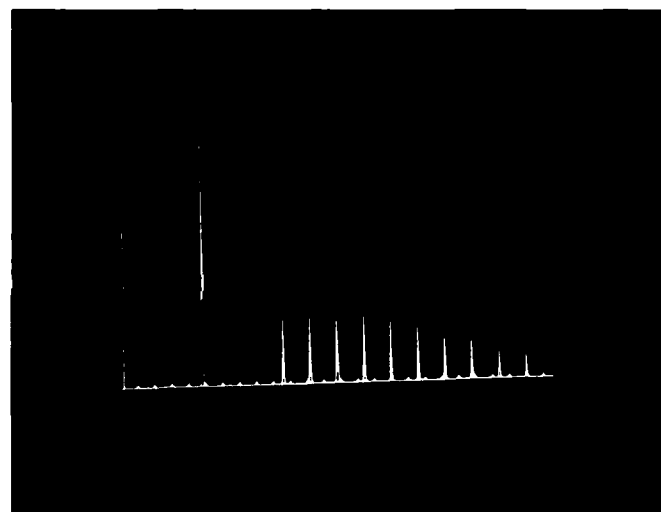
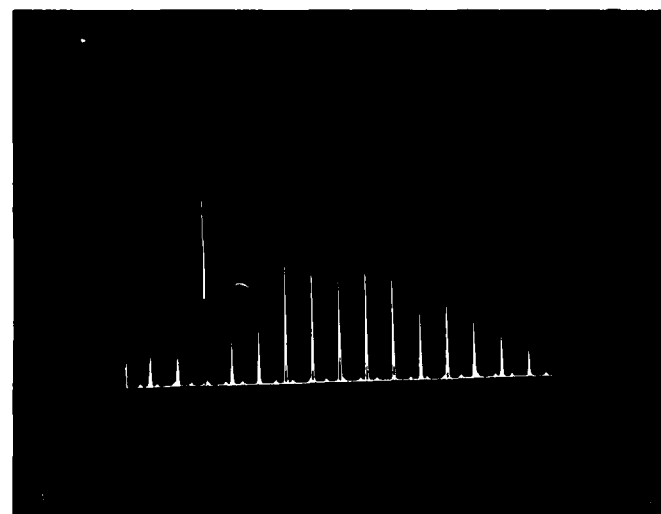


Fig. 2c Subimage constructed  
(two shown above) and the  
histogram of the constructed  
image (right).



### 3. The Maximum Likelihood Estimation

By assuming a first-order autoregressive model, we followed closely the mathematical development in Ref. 2. The maximum likelihood estimate for the regions is obtained by the minimization of the expression (10) in Ref. 2. Fig. 4 shows the maximum likelihood (ML) segmentation obtained by assigning pixels to white and black texture type 1 and 2 respectively. This result is very close to Fig. 2 (b) of Ref. 2 which employed a higher order autoregressive model as described in the previous section. Our first-order model made use of our earlier work reported in Ref. 3.

### 4. The Maximum a Posteriori Estimation

The maximum a posteriori estimation which uses the combination of the Markov chain with a Gaussian autoregressive process is to minimize the expression (11) in Ref. 2. To begin with, the image segmentation is considered as an assignment of the pixels to 0 and 1. Such an assignment for a point  $(n, m)$  is referred to as its "state". The state interdependence complicates the computation of the estimate and an iterative procedure must be employed. The transition probability that plays a major role in the maximum a posteriori (MAP) segmentation is counted from the numbers of state assignment of 0 and 1 for a  $5 \times 5$  neighborhood. It is noted that MAP segmentation starts with the ML segmentation result. For clarity the sequence of lineprinter outputs is shown in Fig. 5. They include the ML segmentation (Fig. 5a), Iteration 1 (Fig. 5b), Iteration 4 (Fig. 5c), Iteration 7 (Fig. 5d) and Iteration 11 (Fig. 5e). At Iteration 11, the small false region in the upper left corner starts to disappear. It is noted that such false region was not

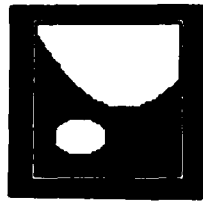


Fig. 3 Ideal segmentation

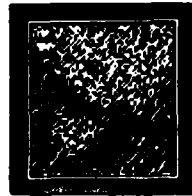


Fig. 4 ML segmentation



Fig. 6a MAP segmentation with 14 iterations



Fig. 6b Smoothing of Fig. 6a

Note: Learning samples for ML and MAP segmentations are based on 480 pixels selected from each type of texture. The following are parameters computed for each texture. Type 1 has mean of 108 and Type 2 has mean of 81.

$\mu(1) = 108$

$\mu(2) = 81$

$\sigma(1) = 0.0000$

$\sigma(2) = 0.0000$

$\sigma(1) = 0.0000$

$\sigma(2) = 0.0000$

$\sigma(1) = 0.0000$

$\sigma(2) = 0.0000$

$\sigma(1) = 0.0000$

$\sigma(2) = 0.0000$

Fig. 5a  
ML  
segmentation

010111000101011011001110011100101101101100010011110001010  
01011111011011011111111100110010001011111011011110111011001  
011111110001000011100100011110110110011111010011110100111000  
0100111000101101111101110001011000110110101110010001000110111  
010101100001100111111110101111001000101001100100100100010101  
000101100011100011010111111111000111101111011101110010101011011  
01111011010001101010100101100101001101101101111011101110011  
0101111111100000101011110010010110000101101011001011110010010  
010111110110111111101111111011110000110111101000101110011100  
000011101101101011011001111111110010101010100100111101000111111  
00010010000011101010010001111011111011101100010111010111101101  
0000101101011110011100111001111100111110001110110110100011101011  
00011001111111111010110111101111011101110011100110000000101111  
00010011110010101001111111110001011111111100111010111110111110  
00110010111010111011110010100010101110011101111001000101111  
0010010001101011111110111010000101011100110001111101111011  
0001011011010011101011101110011100111001101111011100111001110  
001110011011110111101110011100000011010111110100100101101110  
0100000111001010111101111011110010111101111101110001100011110  
0100011010001111110101111111110110011100011111110100011100  
01000010001100111010001110110110101010111110110001001000111100  
0000110000001101001110111011011111011110001100000110111011101  
000001000011110100111101101111111111101110110011110101101110  
00001000000000011010110001111110101010111101011001001110100  
000110000001100100010010001011111010000101011101111101100111  
000110000001001000110011111111011000111010011000010011110111100  
011000001100101110110101100101110110111010001101110101011000  
010000010011000000001000000010000000100110110001001110001000  
0000000010011000000010011111100100001011011110100011000001000  
000000001001100000001001111110011000011111101100011110000  
0100010000001000000001000000001000000001001001100000100000111  
000000100000111101101100000001011100000011000100001000000011  
010000100001000111011001100001101100000011000100001111000010  
0010000000100000111010000110100000010000000100011011100100001000111011  
0010000100100111101000010010110000011111110111101000110001111  
000011100111010001110001110010111110000000010001111111000000  
000001111000111101100101010010001000000000001000001000000010  
0000000011111101111001111101000010000000001001100000001010011  
000000001011110001001111001000011000000001111000000001100000  
000000011110111100001011101001010100000001100011000000100111  
00000110001100101101001110100011000000001100011000000101100  
00111000111001111000110011100110011000000011000100001000111000  
011000001110011110111010110001100000000010001100010000011000  
011110000000100011010111010001110000000000001000110010001100  
0110100000011011101000110000111000100011000111001100010001000  
0101000000100101111111100000010010110010111110000100010010000  
011010000010001100000000111010101001100010100001100010001000  
000101001000000100000001100000001100000001000000100000010000  
0001010010000001000000011000000011000000100000010000001000000  
000101001000000100000001100000011000000100000010000001000000111111

Fig. 5b  
Iteration 1  
MAP  
segmentation



[illegible]





removed in Ref. 2 even after 16 iterations. At Iteration 14, the Tektronix display is shown in Fig. 6a which is then smoothed by using a procedure due to Sklansky with the result shown in Fig. 6b. The percentage segmentation error of Fig. 6b is computed as 3.96%. It is important to note that for all linear estimation methods considered in this report, the supervised learning samples are selected from regions of type 1 and type 2.

#### 5. Fisher's Linear Discriminant

Our initial effort of using Fisher's linear discriminant follows the procedure employed in Ref. 4. The features are the gray levels of the pixels. Fig. 7a is a tabulation of the scattered matrices for Type 1(target) and Type 2 (background) textures. The two scatter matrices are nearly proportional by a factor of 3. The Fisher's linear discriminant simply cannot classify the two types of textures. A new feature selection procedure is used that extracts three features  $x(i)$ ,  $i=1,2,3$  for a  $3 \times 3$  neighborhood

|   |   |   |
|---|---|---|
| A | B | C |
| D | E | F |
| G | H | I |

by computing

$$x(1) = E$$

$$x(2) = (A+B+C+D+E+F+G+H+I)/9$$

$$x(3) = (B+D+F+H) - 4E$$

The learning samples are taken from a  $24 \times 28$  area with each type of texture region. With each  $3 \times 3$  neighborhood represented by the feature vector  $(x(1), x(2), x(3))$ , the two scatter matrices as tabulated in Fig. 7b are quite different. The Fisher's linear discriminant can then successfully segment the subimage as shown in

**Abstract**

\*\*\* SCANNING IN PROGRESS \*\*\*

0.4401E+05 0.1870E+09 0.1012E+10  
0.1452E+09 0.4495E+09 0.2091E+09  
0.1012E+09 0.4410E+09 0.2411E+09

2000 年 1 月 1 日 至 2000 年 12 月 31 日止

|   |    |    |    |    |   |    |    |    |    |    |   |     |    |
|---|----|----|----|----|---|----|----|----|----|----|---|-----|----|
| 0 | 6  | 35 | 4  | 05 | 0 | 27 | 1  | 20 | 09 | 0  | 1 | 750 | 09 |
| 0 | 27 | 1  | 20 | 09 | 0 | 6  | 35 | 4  | 05 | 0  | 1 | 750 | 09 |
| 0 | 11 | 1  | 01 | 09 | 0 | 2  | 68 | 0  | 04 | 09 | 0 | 2   | 68 |

2000 年 12 月 10 日

[illegible]

Fig. 7b

## REFERENCES

11. 00028  
 11. 00029  
 11. 00030

THE UNIVERSITY OF CHICAGO PRESS

[illegible]

Figure 1. The effect of the concentration of the  $\text{H}_2\text{O}_2$  solution on the amount of the  $\text{H}_2\text{O}_2$  consumed in the reaction of the  $\text{H}_2\text{O}_2$  with the  $\text{Fe}^{2+}$  ion.

|  |   |   |
|--|---|---|
| 1. $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$  | 2. $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$   | 3. $\frac{1}{2} \times \frac{1}{5} = \frac{1}{10}$  |
| 4. $\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$ | 5. $\frac{1}{2} \times \frac{1}{7} = \frac{1}{14}$  | 6. $\frac{1}{2} \times \frac{1}{8} = \frac{1}{16}$  |
| 7. $\frac{1}{2} \times \frac{1}{9} = \frac{1}{18}$ | 8. $\frac{1}{2} \times \frac{1}{10} = \frac{1}{20}$ | 9. $\frac{1}{2} \times \frac{1}{11} = \frac{1}{22}$ |

© 2004 Blackwell Publishing Ltd, *Journal of Internal Medicine* 255: 117–127

|  |   |  |
|--|---|--|
| (a) $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i \\ 0 & 1 \end{pmatrix}$ | (b) $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ 0 & 1 \end{pmatrix}$ | (c) $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ i & 1 \end{pmatrix}$  |
| (d) $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ -i & 1 \end{pmatrix}$ | (e) $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$ | (f) $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i \\ i & 1 \end{pmatrix}$ |

10-11-11 11:11:11

[illegible]



Fig. 8a Segmentation by Fisher's linear discriminant

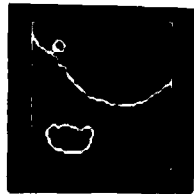


Fig. 8b Sobel gradient operation on Fig. 8a. (threshold is 3.162)



Fig. 8c Robert gradient operation on Fig. 8a. (threshold is 1)

Fig. 8a. The small false region, however, cannot be removed. The results of Sobel and Robert gradient operations on Fig. 8a are shown in Fig. 8b and Fig. 8c respectively. Due to the false region, the percentage segmentation error of Fig. 8a is 4.64%. The Fisher's linear discriminant requires slightly more computation time because of the scatter matrix and feature projection computations.

As a concluding remark, it is interesting to note that while both the maximum a posteriori estimation and the Fisher's linear discriminant are feasible linear estimation methods for image analysis, they employ quite different procedures for utilizing the statistical contextual information.

Acknowledgment We thank Dr. Therrien for helpful communications. The results reported in Sections 3 and 4 are due to R. H. Wu and the results reported in Section 5 are due to C. Yen.

#### References

1. C. H. Chen, "A comparative evaluation of statistical image segmentation techniques," Technical Report SMU-EE-TR-81-3, January 1981.
2. C. W. Therrien, "Linear filtering models for texture classification and segmentation," Proc. of the 5th International Conference on Pattern Recognition, pp. 1132-1135, Dec. 1980.
3. C. H. Chen and J. Chen, "A comparison of image enhancement techniques," Technical Report SMU-EE-TR-8, Dec. 1979. Also appeared in the Proc. of 1980 IEEE International Conference on Cybernetics and Society.
4. C. H. Chen and C. Yen, "Statistical image segmentation using Fisher's linear discriminant," submitted for publication, 1980.

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

| REPORT DOCUMENTATION PAGE   |                       | READ INSTRUCTIONS<br>BEFORE COMPLETING FORM                 |
|---|-----------------------|---|
| 1. REPORT NUMBER  | 2. GOVT ACCESSION NO. | 3. RECIPIENT'S CATALOG NUMBER                               |
|   | AD-A094281            |   |
| 4. TITLE (and Subtitle)   |                       | 5. TYPE OF REPORT & PERIOD COVERED                          |
| (6) Some Experimental Results on Linear Estimation for Image Analysis.  |                       | (9) Technical Report.                                       |
| 6. AUTHOR(s)  |                       | 7. PERFORMING ORG. REPORT NUMBER                            |
| (10) C. H./Chen, Ron-Hwang/Wu Chih-sung/Yen   |                       | (14) SMU-EE-TR-81-4   |
| 8. PERFORMING ORGANIZATION NAME AND ADDRESS   |                       | 9. CONTRACT OR GRANT NUMBER(s)                              |
| Electrical Engineering Department<br>Southeastern Massachusetts University<br>North Dartmouth, MA 02747   |                       | (15) N00014-79-C-0494                                       |
| 10. CONTROLLING OFFICE NAME AND ADDRESS   |                       | 11. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS |
| Statistics and Probability Program<br>Office of Naval Research, Code 436<br>Arlington, VA 22217   |                       | NR 042-422  |
| 12. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)   |                       | 13. REPORT DATE   |
| (12) 191  |                       | 11/28 Jan 88, 1981  |
|   |                       | 14. NUMBER OF PAGES   |
|   |                       | 19  |
|   |                       | 15. SECURITY CLASS. (of this report)                        |
|   |                       | Unclassified  |
|   |                       | 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE                  |
| 16. DISTRIBUTION STATEMENT (of this Report)   |                       |   |
| APPROVED FOR PUBLIC RELEASE: DISTRIBUTION UNLIMITED.  |                       |   |
| 17. DISTRIBUTION STATEMENT of the abstract entered in Block 20, if different from Report)   |                       |   |
| 18. SUPPLEMENTARY NOTES   |                       |   |
| 19. KEY WORDS (Continue on reverse side if necessary and identify by block number)  |                       |   |
| Maximum likelihood estimation<br>Maximum a posteriori estimation<br>Transition probability<br>Fisher's linear discriminant  |                       |   |
| 20. ABSTRACT (Continue on reverse side if necessary and identify by block number)   |                       |   |
| Based on a constructed textured subimage, an experimental comparison is made on the three linear estimation methods for image segmentation, namely the maximum likelihood estimation, the maximum a posteriori estimation and the Fisher's linear discriminant. The last two methods are shown to be feasible for effective image segmentation. |                       |   |

DD FORM 1473 EDITION OF NOV 65 IS OBSOLETE

U.S. GOVERNMENT PRINTING OFFICE

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

407932

911

DATE  
FILMED  
- 8

